On the explorative behavior of *MAX–MIN* Ant System

Additional results

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Abstract Analyzing the behavior of stochastic procedures is generally recognized to be relevant. A possible way for doing so consists in observing the exploration performed. A formalization in this sense is proposed in the paper *On the explorative behavior of MAX–MIN Ant System* : A method for studying this aspect regardless the type of approach used is defined and tested. The consequent measure of exploration is applied to *MAX–MIN* Ant System: The impact of the values of the parameters on the exploration is assessed. The TSP is considered as benchmark. Moreover, the conclusions drawn are put in relation with the indications provided by the average λ-branching factor. Finally, some relations are pointed out between exploration and performance of the algorithm.
Owing to lack of space, few results are reported in *On the explorative behavior of MAX–MIN Ant System* . A deeper experimental analysis is reported here.

1 Experimental analysis

The impact of the values of the parameters on the exploration performed by *MAX–MIN* Ant System is analyzed. The ACOTSP program implemented by Thomas Stützle is considered. The code has been released in the public domain and is available for free download on www.aco-metaheuristic.org/aco-code/. The stopping criterion considered in this analysis consists in the fulfillment of 20000 objective function evaluations. At this early stage, no local search procedure is ap-

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plied. The code used for computing the exploration is available on the web page http://www.paola.pellegrini.it.

One hundred TSP instances are used. They are generated through `portgen`, the instance generator adopted in the DIMACS TSP Challenge. In particular, they consist of two dimensional integer-coordinate cities distributed in a square of size $10^6 \times 10^6$. In half of the instances cities are grouped in clusters, in the others they are uniformly distributed. Each instance includes 50 nodes. They are available on the web page http://www.paola.pellegrini.it. The set of experiments described below have been performed also on one instance with 100 and 200 nodes. The results appear qualitatively equivalent. They are reported in Appendix 2.

MAX–MIN Ant System is run varying the values of the parameters. According to the literature these values have an impact on the exploration. The following analysis aims at observing this difference of exploration during the evolution of a run. The solutions are grouped in sets of cardinality equal to 50, and the exploration is computed over these sets. The conclusions drawn are considered in the light of the results obtained by studying the trend of the average λ-branching factor computed in the same horizon.

The parameters analyzed are α, β, ρ, m. The values $\alpha = 1, \beta = 1, \rho = 0.02, m = 50$ are considered as reference. Then, one parameter at a time is varied. The values used for parameters are reported the following: $\alpha \in \{1,2,3,4\}, \beta \in \{1,2,3,4\}, \rho \in \{0.02, 0.05, 0.2, 0.7\}, m \in \{20,30,50,70\}$.

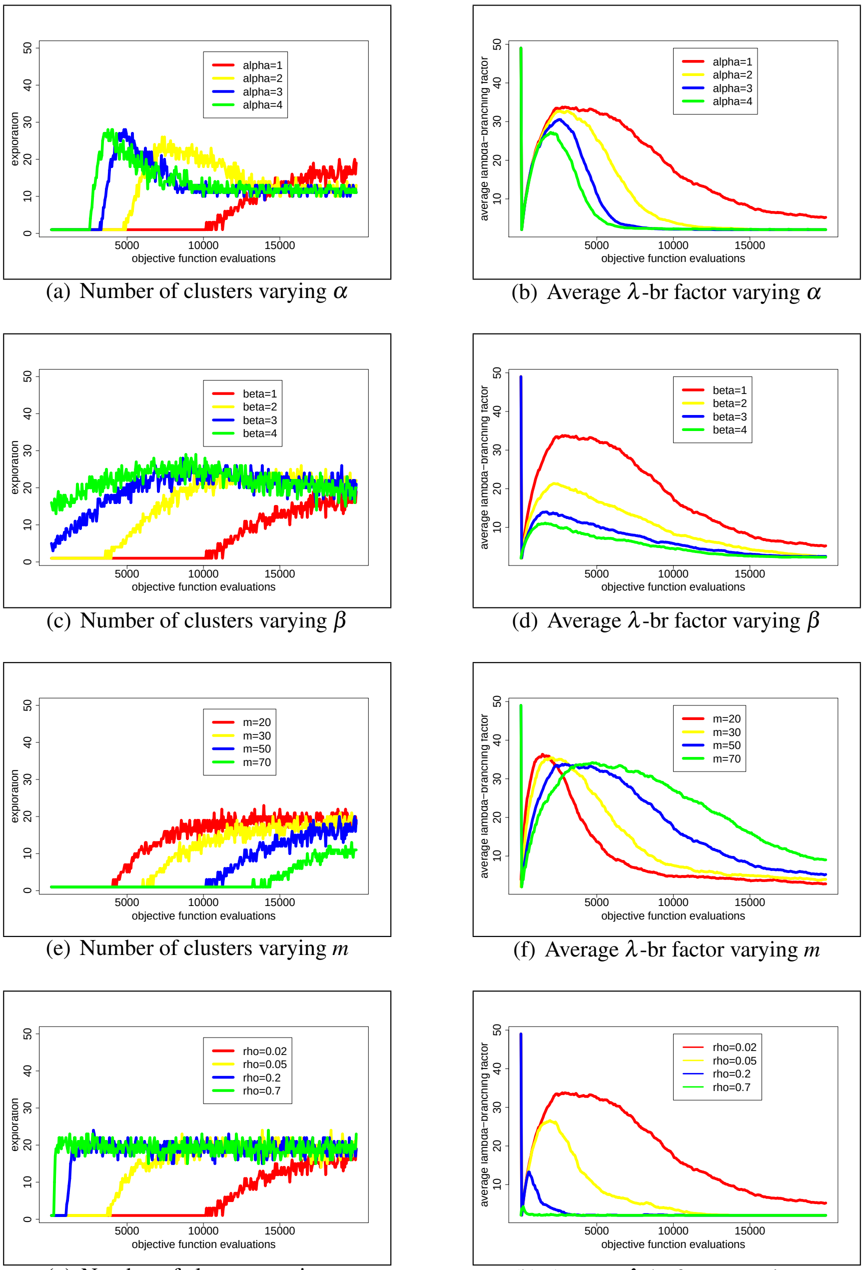
In figures 1(a), 1(c), 1(e), 1(g), the exploration is reported for the different values of the parameters.

It can be observed that an increase of the value of α (figure 1(a)) has three main effects: First of all a significant level of exploration is achieved earlier in the run, as well as its peak. Moreover, the higher the value of α, the higher this maximum level. It has to be remarked that an exploration equal to 1 characterizes the beginning of the run whatever value is set for α. In this phase the algorithm is moving in the search space in a quite random fashion. As a consequence the solutions visited are likely to be different, as they would be if a random search was applied. In this latter case, two solutions differing for all the edges have a distance of $\sqrt{2 \cdot 50 \cdot \left(\frac{1}{50}\right)^2} = 0.2$ which is much lower than ε_{10}. This phase will be referred to as *random–like*. Until the probability of some edges passes a certain threshold the behavior remains substantially the same and the exploration is equal to 1.

For what β is concerned (figure 1(c)) a very similar trend can be noted. The main difference consists in the fact that the *random–like* phase is completely absent when $\beta = 3$ and $\beta = 4$. In general, moreover, the peaks of exploration are much less pronounced, and they are reached after a smoother ascent.

This dependence of the exploration from α and β is coherent with the role of these parameters: Consider the ratio between the probabilities associated to two edges i and j at a specific time instant:

$$\frac{P_i}{P_j} = \left[\frac{\tau_i}{\tau_j}\right]^{\alpha} \left[\frac{\eta_i}{\eta_j}\right]^{\beta}. \tag{1}$$



(a) Number of clusters varying α

(b) Average λ-br factor varying α

(c) Number of clusters varying β

(d) Average λ-br factor varying β

(e) Number of clusters varying m

(f) Average λ-br factor varying m

(g) Number of clusters varying ρ

(h) Average λ-br factor varying ρ

Fig. 1 Number of clusters versus average λ-branching factor

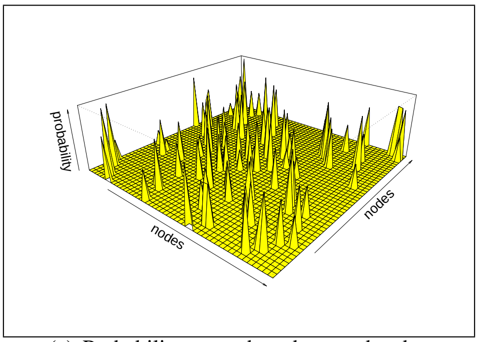
Suppose, with no loss of generality, that $\eta_i > \eta_j$. It is evident that, the further the value of (1) from 1, the stronger the attractiveness of edge i with respect to j. During a run of the algorithm, the only variable element in (1) is the ratio between pheromone values. A high value of α implies that a slight change in this ratio, going from slightly below to slightly above 1 (or viceversa), implies a strong variation of the ratio between the probabilities. A similar reasoning can be made for β: a high value of this parameter amplifies a lot any variation of τ_i/τ_j. As a consequence, the lower these parameters, the less extreme the short term variation of the ratio between probabilities. It follows that, the evaporation rate being equal, the duration of *random–like* phase is inversely proportional to the values of these parameters.

Figure 1(e) shows that the greater the number of ants, the longer the *random–like* phase. This is due to the fact that given the number of objective function evaluations, the higher the value of m the fewer the pheromone updates performed.

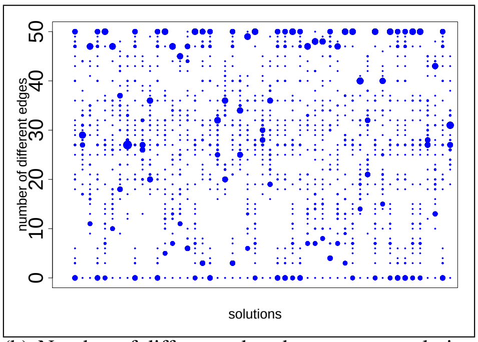
Finally, ρ implies the same trend depicted for α and β: When ρ increases the duration of the *random–like* phase is shorter. Peaks are much less stressed than in the case of α. An actual decrease of exploration after the maximum is reached is not even evident. An explanation of this effect can be found in the pheromone update rule. In *MAX–MIN* Ant System pheromone evaporates on all the edges, and is deposited only on those belonging to the best so far (or iteration best) solution. It implies that if the evaporation rate is large, even after few updates the differences in the pheromone trails may be very strong.

In figures 1(b), 1(d), 1(f) and 1(h) the average λ-branching factor is reported. For all the values of the parameters, it increases for a while and then starts descending, more or less steeply. This reflects the observations made on the exploration: The period in which the average λ-branching factor is high, and then in which the pheromone is almost uniformly distributed on the edges, corresponds to the *random–like* phase: The algorithm has not yet defined any real difference among the solutions.

A point that deserves a deeper analysis is the fact that when the average λ-branching factor reaches a very low value, the corresponding exploration appears to be quite high. In this framework the nature of the two measures needs to be taken into account: The former concerns only the pheromone distribution. The latter considers both the probability distribution (which is indeed connected to the pheromone trails) and the solutions actually visited. Figure 2 aims at underlying this difference. The last 50 solutions generated in the run with $m = 50$, $\alpha = 1$, $\beta = 1$ and $\rho = 0.7$ are considered. In figure 2(a) the probability associated to each node to node edge is presented. It is evident that the distribution is coherent with a very low average λ-branching factor: not more that a couple of edges incident on each node have high probability. On the other hand, figure 2(b) investigates the difference among the solutions found. In particular, they are considered here in a TSP-wise sense: only the permutations matter, while probabilities are completely neglected. The number of different edges between one solution and each other is reported. For each pair $(\hat{x}, \hat{y})$, the size of the bullet used is proportional to the number of solutions differing from solution $\hat{x}$ for $\hat{y}$ edges. It is evident that, despite the very concentrated pheromone trails, the solutions visited are far from being always the same. This is due to the fact



(a) Probability on each node to node edge

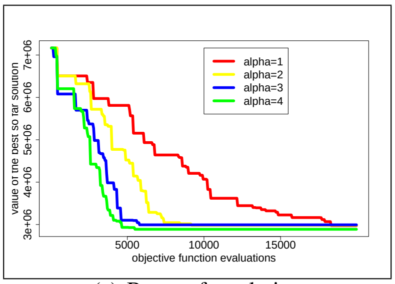


(b) Number of different edges between one solution and each other

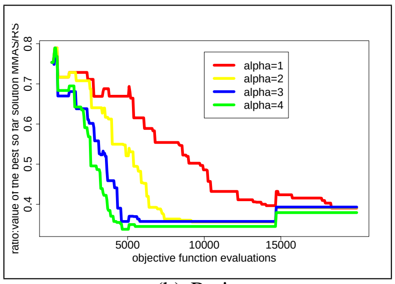
Fig. 2 Observations on the last 50 solutions visited using parameters $m = 50, \alpha = 1, \beta = 1, \rho = 0.7$

that it may well happen that, given a partial solution generated, an ant is in a node and all the feasible edges have very similar low probability: The edges with high pheromone level would bring the ant to already visited nodes. In this case several different solutions are likely to be visited.

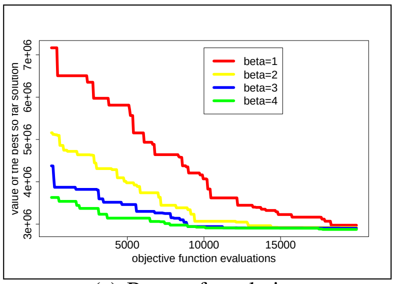
This sort of contradiction between the indication of the two measures is due to the fact that, although they are connected to the parallel concepts of exploration and stagnation, they aim at different objectives. The evaluation of the number of clusters of solutions visited is finalized at observing the behavior of the algorithm. Instead, the average λ-branching factor is oriented to the identification of the moment in which the pheromone needs to be re-initialized during a run. In this sense, if the pheromone trail is concentrated on very few edges, two types of solutions are likely to be generated: On the one hand it is possible that the indications of the pheromone are perfectly followed. The consequence is the construction of an already visited solution, which is clearly a waste of time. On the other hand, it may be unfeasible to use some of the reinforced edges. An partially random solution is, then, generated. It is much more convenient, and coherent with the spirit of the algorithm, to re-initialize the pheromone trail. In this sense, the average λ-branching factor is a



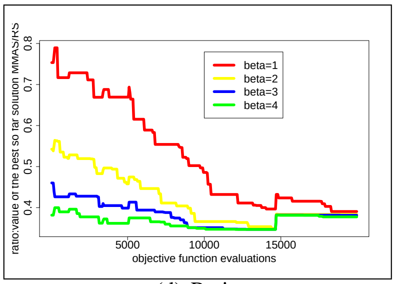
(a) Best so far solution



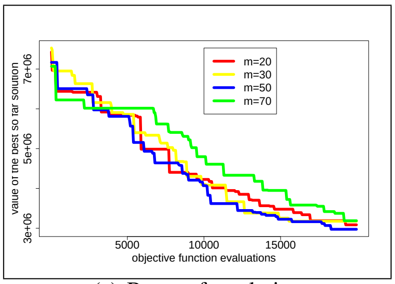
(b) Ratio



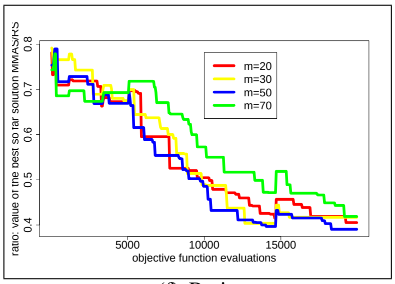
(c) Best so far solution



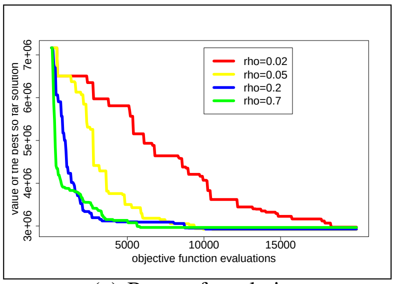
(d) Ratio



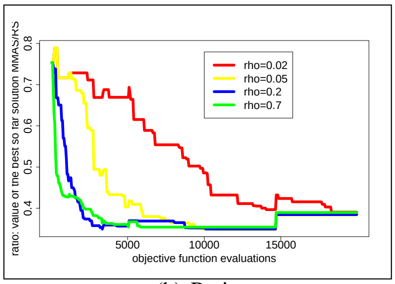
(e) Best so far solution



(f) Ratio



(g) Best so far solution



(h) Ratio

Fig. 3 Value of the best so far solution and ratio with respect to random search fundamental measure for the effectiveness of *MAX–MIN* Ant System. Nonetheless, even if it has a close connection with stagnation, it has to be remarked that the assessment of the number of clusters represents much more accurately the actual behavior of the procedure.

Table 1 Thousands of objective function evaluations after which the exploration reaches its maximum value. Between parenthesis, corresponding value of exploration.

inst	configuration															
	$\beta=1, \rho=0.02, m=50$				$\alpha=1, \rho=0.02, m=50$				$\alpha=1, \beta=1, \rho=0.02$				$\alpha=1, \beta=1, m=50$			
	$\alpha=$				$\beta=$				$m=$				$\rho=$			
	1	2	3	4	1	2	3	4	20	30	50	70	0.02	0.05	0.2	0.7
1	19(20)	7(26)	5(28)	4(28)	19(20)	16(26)	9(28)	9(29)	14(23)	19(21)	19(20)	20(13)	19(20)	14(24)	3(24)	14(24)
2	18(21)	7(28)	4(33)	4(33)	18(21)	10(29)	7(32)	6(33)	17(25)	16(23)	18(21)	20(21)	18(21)	12(24)	2(24)	5(24)
3	17(20)	8(30)	5(31)	4(35)	17(20)	16(28)	8(30)	6(32)	18(24)	16(23)	17(20)	20(15)	17(20)	11(24)	5(24)	11(25)
4	20(18)	8(27)	5(30)	4(31)	20(18)	15(27)	10(30)	6(31)	18(23)	18(23)	20(18)	19(15)	20(18)	14(23)	2(24)	1(24)
5	18(17)	7(27)	4(28)	4(31)	18(17)	14(27)	10(30)	8(31)	15(22)	19(23)	18(17)	18(13)	18(17)	16(24)	5(24)	0(24)
6	18(16)	8(24)	5(27)	4(31)	18(16)	14(25)	10(28)	12(29)	18(23)	19(23)	18(16)	20(12)	18(16)	17(22)	19(25)	3(24)
7	20(19)	8(27)	5(30)	4(28)	20(19)	19(27)	10(29)	10(30)	14(22)	20(22)	20(19)	20(10)	20(19)	14(24)	10(24)	0(24)
8	19(15)	8(27)	5(29)	4(30)	19(15)	12(26)	9(29)	7(33)	18(22)	16(22)	19(15)	19(13)	19(15)	11(23)	8(25)	0(24)
9	17(21)	6(29)	5(36)	4(34)	17(21)	11(29)	6(31)	6(33)	14(24)	17(24)	17(21)	18(18)	17(21)	12(25)	6(24)	0(25)
10	19(20)	7(27)	5(30)	4(30)	19(20)	13(27)	9(30)	7(30)	19(23)	16(23)	19(20)	19(14)	19(20)	16(25)	3(23)	0(24)
11	20(14)	7(25)	5(32)	4(30)	20(14)	17(26)	13(29)	9(28)	19(22)	20(19)	20(14)	20(11)	20(14)	16(24)	6(24)	15(25)
12	19(20)	7(24)	5(28)	4(27)	19(20)	15(25)	9(27)	7(28)	17(22)	17(22)	19(20)	20(11)	19(20)	12(23)	10(23)	3(24)
13	18(18)	7(27)	5(28)	4(30)	18(18)	13(27)	8(29)	7(30)	16(23)	18(22)	18(18)	20(17)	18(18)	12(23)	3(23)	7(24)
14	16(21)	6(26)	5(30)	4(32)	16(21)	11(26)	10(30)	6(31)	16(23)	17(22)	16(21)	19(17)	16(21)	11(23)	5(23)	0(24)
15	18(21)	7(28)	4(31)	4(32)	18(21)	12(28)	6(31)	7(32)	13(24)	15(22)	18(21)	19(18)	18(21)	11(24)	3(25)	1(26)
16	19(21)	7(27)	5(31)	4(31)	19(21)	13(27)	11(29)	5(30)	14(23)	16(23)	19(21)	19(18)	19(21)	11(24)	2(24)	1(23)
17	19(9)	8(27)	5(28)	4(30)	19(9)	14(26)	10(28)	8(29)	16(21)	18(19)	19(9)	20(7)	19(9)	17(22)	3(23)	0(23)
18	20(22)	7(28)	5(30)	4(32)	20(22)	12(26)	10(29)	6(32)	18(23)	18(24)	20(22)	20(17)	20(22)	8(24)	6(24)	0(25)
19	20(18)	8(26)	5(28)	4(31)	20(18)	17(27)	11(30)	11(32)	19(22)	13(21)	20(18)	19(15)	20(18)	16(23)	2(24)	17(24)
20	18(24)	7(31)	5(36)	4(34)	18(24)	15(29)	10(33)	5(35)	15(24)	18(24)	18(24)	18(20)	18(24)	11(25)	3(24)	0(25)
21	17(19)	7(30)	5(31)	4(34)	17(19)	10(27)	10(31)	9(33)	19(22)	18(25)	17(19)	19(13)	17(19)	16(24)	17(24)	11(25)
22	18(21)	6(28)	5(34)	3(35)	18(21)	12(29)	8(32)	7(33)	17(25)	17(25)	18(21)	19(18)	18(21)	10(25)	4(25)	1(24)
23	17(24)	6(31)	5(37)	4(37)	17(24)	13(33)	8(37)	5(39)	12(27)	12(26)	17(24)	19(23)	17(24)	10(27)	1(27)	0(25)
24	19(20)	7(29)	5(33)	4(31)	19(20)	17(30)	10(33)	7(36)	19(24)	18(24)	19(20)	20(18)	19(20)	10(24)	2(24)	0(24)
25	20(21)	6(29)	4(31)	4(31)	20(21)	13(26)	9(27)	5(27)	15(24)	16(22)	20(21)	20(17)	20(21)	11(24)	1(23)	0(24)
26	19(19)	7(27)	5(26)	4(32)	19(19)	16(26)	18(28)	10(30)	20(23)	16(22)	19(19)	20(14)	19(19)	14(23)	15(24)	0(24)
27	16(20)	8(28)	5(29)	4(32)	16(20)	17(28)	12(30)	10(32)	17(24)	16(23)	16(20)	19(16)	16(20)	14(25)	14(24)	1(26)
28	18(18)	7(28)	5(29)	4(30)	18(18)	15(26)	19(28)	6(29)	20(23)	20(22)	18(18)	18(13)	18(18)	19(23)	16(26)	4(24)
29	19(19)	7(27)	5(30)	4(32)	19(19)	11(28)	9(30)	6(32)	19(24)	19(23)	19(19)	19(15)	19(19)	15(25)	3(26)	0(24)
30	19(24)	6(29)	4(31)	4(37)	19(24)	12(30)	9(32)	6(35)	10(24)	18(25)	19(24)	20(21)	19(24)	10(24)	2(24)	3(25)
31	18(20)	8(26)	5(28)	4(28)	18(20)	14(26)	11(28)	8(30)	15(24)	20(23)	18(20)	20(13)	18(20)	18(24)	8(25)	0(25)
32	19(22)	6(29)	4(29)	3(31)	19(22)	13(27)	11(30)	7(31)	14(24)	18(24)	19(22)	19(18)	19(22)	15(24)	9(25)	0(25)
33	19(18)	7(25)	5(29)	4(35)	19(18)	14(26)	7(28)	8(31)	15(23)	19(21)	19(18)	19(11)	19(18)	12(23)	2(23)	1(25)
34	20(22)	7(31)	5(32)	4(33)	20(22)	12(30)	7(31)	9(33)	15(25)	14(24)	20(22)	19(20)	20(22)	10(25)	3(24)	0(25)
35	19(19)	7(27)	6(29)	4(29)	19(19)	14(27)	13(29)	10(31)	20(26)	18(24)	19(19)	18(13)	19(19)	18(25)	4(24)	1(24)
36	19(14)	8(28)	5(29)	4(29)	19(14)	16(26)	16(29)	11(29)	20(22)	18(20)	19(14)	19(9)	19(14)	15(23)	15(24)	3(24)
37	20(20)	7(29)	5(31)	4(36)	20(20)	14(28)	7(30)	6(33)	12(22)	17(22)	20(20)	19(16)	20(20)	11(24)	2(24)	1(25)
38	19(20)	8(28)	5(31)	4(27)	19(20)	12(29)	8(31)	7(32)	15(23)	18(23)	19(20)	20(16)	19(20)	19(24)	4(25)	4(24)
39	17(21)	6(28)	5(34)	4(34)	17(21)	11(27)	8(30)	5(32)	17(25)	15(23)	17(21)	20(16)	17(21)	11(24)	3(24)	3(25)
40	16(28)	6(38)	4(39)	4(42)	16(28)	7(36)	6(39)	4(41)	7(27)	13(28)	16(28)	17(28)	16(28)	5(27)	1(27)	0(27)
41	19(21)	7(29)	5(32)	4(31)	19(21)	12(28)	7(31)	7(32)	12(24)	20(24)	19(21)	19(18)	19(21)	13(25)	2(25)	16(24)
42	19(22)	7(32)	5(35)	4(38)	19(22)	14(31)	9(32)	7(32)	15(25)	14(24)	19(22)	20(23)	19(22)	10(25)	2(25)	0(25)
43	17(22)	8(29)	5(30)	4(31)	17(22)	12(31)	11(32)	4(33)	17(24)	16(24)	17(22)	18(19)	17(22)	11(25)	2(24)	0(24)
44	19(20)	7(27)	5(35)	4(33)	19(20)	14(28)	8(30)	8(31)	18(25)	19(26)	19(20)	20(16)	19(20)	10(24)	4(24)	13(24)
45	19(22)	8(28)	5(33)	4(33)	19(22)	15(29)	11(29)	7(30)	14(24)	19(23)	19(22)	19(15)	19(22)	11(24)	10(24)	1(24)
46	16(18)	8(27)	5(31)	4(30)	16(18)	12(26)	14(29)	7(29)	19(24)	18(20)	16(18)	20(15)	16(18)	16(24)	2(24)	1(24)
47	18(20)	7(28)	5(30)	4(31)	18(20)	11(27)	9(32)	6(32)	14(24)	18(23)	18(20)	20(16)	18(20)	17(25)	2(25)	0(24)
48	19(24)	7(29)	5(33)	4(35)	19(24)	16(31)	9(33)	5(35)	13(25)	14(24)	19(24)	17(20)	19(24)	10(25)	2(25)	2(27)
49	19(20)	8(27)	5(27)	4(33)	19(20)	15(26)	9(28)	12(29)	16(23)	16(23)	19(20)	20(11)	19(20)	12(23)	4(24)	0(24)
50	19(21)	6(26)	5(30)	4(34)	19(21)	14(27)	11(31)	8(32)	17(23)	20(22)	19(21)	20(14)	19(21)	10(23)	3(24)	7(24)
51	19(18)	7(27)	5(34)	4(33)	19(18)	11(26)	9(30)	8(31)	19(23)	19(22)	19(18)	18(13)	19(18)	15(24)	5(25)	1(24)
52	20(20)	8(28)	5(29)	4(27)	20(20)	12(26)	8(28)	6(29)	12(23)	19(23)	20(20)	19(12)	20(20)	11(24)	4(24)	0(26)
53	20(22)	7(28)	5(28)	4(36)	20(22)	11(27)	7(29)	5(33)	17(23)	17(23)	20(22)	20(17)	20(22)	12(24)	2(23)	0(24)
54	20(18)	8(27)	5(28)	4(30)	20(18)	13(27)	7(30)	4(30)	19(23)	15(22)	20(18)	18(15)	20(18)	12(24)	9(24)	1(25)
55	18(23)	7(31)	5(35)	4(36)	18(23)	13(31)	8(34)	6(35)	19(24)	15(24)	18(23)	20(20)	18(23)	12(24)	3(25)	3(25)
56	18(20)	8(25)	5(29)	4(29)	18(20)	16(25)	8(27)	6(29)	20(24)	19(22)	18(20)	19(14)	18(20)	14(23)	2(25)	16(24)
57	18(22)	7(31)	5(35)	4(35)	18(22)	11(29)	11(34)	8(36)	16(24)	12(24)	18(22)	19(18)	18(22)	17(24)	3(25)	1(24)
58	18(25)	6(33)	4(38)	4(40)	18(25)	11(34)	9(36)	7(42)	11(26)	15(25)	18(25)	19(23)	18(25)	12(27)	2(25)	0(27)
59	19(20)	7(26)	5(30)	4(31)	19(20)	15(28)	12(30)	5(30)	12(23)	18(24)	19(20)	19(15)	19(20)	17(24)	2(25)	0(26)
60	18(18)	7(28)	5(31)	4(33)	18(18)	15(26)	11(29)	7(32)	16(24)	16(22)	18(18)	20(14)	18(18)	14(24)	6(25)	1(24)

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inst	configuration: $\beta=1, \rho=0.02, m=50$, $\alpha=$ 1	2	3	4	$\alpha=1, \rho=0.02, m=50$, $\beta=$ 1	2	3	4	$\alpha=1, \beta=1, \rho=0.02$, $m=$ 20	30	50	70	$\alpha=1, \beta=1, m=50$, $\rho=$ 0.02	0.05	0.2	0.7
61	19(21)	8(28)	5(33)	4(33)	19(21)	14(26)	10(29)	7(30)	15(24)	18(24)	19(21)	19(15)	19(21)	14(26)	3(24)	0(25)
62	17(20)	8(28)	5(31)	4(31)	17(20)	16(27)	8(28)	8(29)	19(23)	18(22)	17(20)	19(17)	17(20)	17(25)	3(24)	2(25)
63	19(19)	7(28)	5(29)	4(29)	19(19)	17(27)	16(29)	11(30)	16(24)	20(20)	19(19)	19(15)	19(19)	15(24)	2(24)	4(25)
64	19(23)	7(29)	5(31)	4(34)	19(23)	14(27)	7(30)	5(32)	19(24)	14(22)	19(23)	20(18)	19(23)	19(25)	2(23)	1(25)
65	18(20)	7(27)	5(32)	4(31)	18(20)	12(27)	8(29)	7(30)	18(24)	18(23)	18(20)	20(17)	18(20)	10(23)	6(24)	9(24)
66	19(15)	8(28)	5(32)	4(31)	19(15)	14(26)	11(29)	12(30)	16(21)	20(21)	19(15)	20(10)	19(15)	17(23)	3(24)	0(25)
67	17(24)	7(30)	5(33)	4(35)	17(24)	10(32)	7(34)	7(36)	14(25)	18(25)	17(24)	20(24)	17(24)	10(26)	2(25)	1(26)
68	18(22)	7(27)	4(29)	4(29)	18(22)	10(28)	8(28)	6(30)	17(24)	13(23)	18(22)	19(17)	18(22)	12(24)	2(24)	1(25)
69	20(20)	7(27)	5(31)	4(30)	20(20)	10(28)	9(31)	7(32)	17(24)	18(22)	20(20)	20(18)	20(20)	14(25)	1(24)	0(24)
70	19(24)	7(26)	5(32)	4(29)	19(24)	12(28)	13(30)	6(32)	17(25)	11(23)	19(24)	18(19)	19(24)	10(24)	12(24)	1(24)
71	18(22)	6(26)	5(30)	4(30)	18(22)	10(27)	7(30)	7(33)	15(23)	14(23)	18(22)	19(15)	18(22)	11(24)	2(23)	1(24)
72	19(18)	7(28)	5(28)	4(28)	19(18)	16(26)	11(29)	8(30)	18(22)	18(22)	19(18)	20(11)	19(18)	15(23)	14(24)	0(25)
73	20(23)	6(27)	5(32)	4(34)	20(23)	11(29)	9(32)	6(33)	11(24)	15(24)	20(23)	19(20)	20(23)	12(24)	3(24)	1(25)
74	18(18)	7(28)	5(31)	4(36)	18(18)	16(26)	11(29)	5(31)	17(24)	19(22)	18(18)	20(15)	18(18)	15(24)	3(24)	1(25)
75	19(18)	8(29)	5(28)	4(33)	19(18)	15(25)	14(28)	8(28)	19(23)	16(20)	19(18)	19(8)	19(18)	19(24)	3(24)	0(24)
76	20(18)	7(28)	5(29)	5(32)	20(18)	19(26)	17(29)	9(29)	18(25)	18(20)	20(18)	20(12)	20(18)	18(24)	3(24)	1(24)
77	20(19)	8(30)	5(29)	4(31)	20(19)	13(27)	9(29)	6(30)	18(23)	17(21)	20(19)	19(17)	20(19)	10(24)	3(24)	14(25)
78	20(23)	6(28)	5(31)	4(34)	20(23)	13(28)	8(32)	6(31)	16(24)	18(24)	20(23)	20(20)	20(23)	9(25)	1(24)	5(23)
79	18(21)	7(28)	5(31)	4(33)	18(21)	13(27)	12(30)	6(30)	17(24)	15(22)	18(21)	20(18)	18(21)	15(25)	4(24)	1(25)
80	16(18)	7(28)	5(32)	4(37)	16(18)	15(28)	10(31)	7(31)	20(23)	17(21)	16(18)	18(13)	16(18)	13(24)	2(23)	1(25)
81	18(20)	7(30)	5(32)	5(32)	18(20)	12(29)	10(30)	7(31)	12(23)	19(23)	18(20)	19(19)	18(20)	10(24)	2(25)	0(24)
82	20(23)	7(29)	5(36)	4(35)	20(23)	12(28)	8(30)	7(32)	16(26)	17(24)	20(23)	19(19)	20(23)	12(25)	2(24)	0(25)
83	20(19)	8(29)	5(31)	4(35)	20(19)	17(27)	8(28)	8(29)	20(22)	20(22)	20(19)	20(14)	20(19)	17(25)	9(24)	0(24)
84	17(20)	8(28)	5(30)	4(32)	17(20)	13(27)	8(30)	6(32)	19(24)	14(21)	17(20)	20(16)	17(20)	9(24)	3(24)	2(24)
85	18(19)	7(27)	5(34)	4(36)	18(19)	12(25)	10(31)	8(31)	19(22)	16(21)	18(19)	19(14)	18(19)	14(24)	2(23)	0(25)
86	19(21)	7(28)	5(31)	4(35)	19(21)	15(27)	8(29)	6(31)	17(24)	19(23)	19(21)	20(16)	19(21)	16(25)	3(24)	20(24)
87	18(20)	7(25)	5(31)	4(30)	18(20)	16(26)	9(28)	7(30)	20(23)	20(24)	18(20)	19(16)	18(20)	10(23)	2(24)	14(25)
88	16(19)	7(28)	4(30)	4(34)	16(19)	16(28)	7(30)	5(32)	17(24)	18(23)	16(19)	20(16)	16(19)	13(24)	5(24)	1(24)
89	18(19)	7(27)	5(30)	4(33)	18(19)	20(26)	10(28)	6(29)	12(22)	20(22)	18(19)	19(11)	18(19)	18(24)	2(24)	0(24)
90	19(23)	7(30)	5(32)	4(35)	19(23)	14(29)	12(32)	9(35)	14(24)	18(26)	19(23)	19(19)	19(23)	7(24)	3(24)	18(25)
91	19(18)	8(25)	5(28)	4(31)	19(18)	11(23)	18(28)	8(28)	17(21)	19(20)	19(18)	19(10)	19(18)	18(23)	3(24)	0(23)
92	15(24)	6(31)	4(30)	3(38)	15(24)	9(29)	6(33)	4(35)	13(25)	14(25)	15(24)	19(20)	15(24)	8(24)	3(23)	1(25)
93	20(23)	7(28)	4(32)	3(31)	20(23)	11(29)	7(31)	8(33)	15(25)	16(25)	20(23)	19(20)	20(23)	10(25)	2(25)	0(23)
94	19(21)	7(27)	5(29)	4(30)	19(21)	15(26)	12(29)	9(31)	19(23)	20(21)	19(21)	19(12)	19(21)	12(23)	3(23)	2(24)
95	20(18)	7(26)	5(28)	4(29)	20(18)	14(26)	9(28)	8(28)	18(23)	20(23)	20(18)	19(9)	20(18)	11(22)	15(24)	0(24)
96	19(20)	6(30)	5(34)	4(35)	19(20)	13(28)	10(32)	6(35)	17(25)	18(23)	19(20)	19(17)	19(20)	16(24)	2(24)	1(26)
97	17(20)	8(27)	6(31)	4(29)	17(20)	16(27)	9(27)	8(29)	14(22)	18(22)	17(20)	20(14)	17(20)	13(24)	3(23)	8(24)
98	18(23)	7(28)	5(32)	4(30)	18(23)	15(28)	6(30)	6(34)	14(24)	20(24)	18(23)	20(17)	18(23)	12(24)	2(25)	0(24)
99	20(23)	7(29)	4(34)	4(36)	20(23)	11(28)	9(31)	5(33)	17(23)	19(24)	20(23)	19(17)	20(23)	20(24)	2(24)	0(24)
100	19(21)	7(28)	5(30)	4(33)	19(21)	14(28)	10(31)	7(33)	13(23)	18(23)	19(21)	19(17)	19(21)	10(25)	3(25)	0(25)

Table 2 Thousands of objective function evaluations after which the average λ-branching factor is smaller than 3.

inst	configuration: $\beta=1, \rho=0.02, m=50$, $\alpha=$ 1	2	3	4	$\alpha=1, \rho=0.02, m=50$, $\beta=$ 1	2	3	4	$\alpha=1, \beta=1, \rho=0.02$, $m=$ 20	30	50	70	$\alpha=1, \beta=1, m=50$, $\rho=$ 0.02	0.05	0.2	0.7
1	20	11	7	6	20	18	15	13	19	20	20	20	20	11	3	0
2	20	10	8	7	20	20	17	17	17	17	20	20	20	9	2	0
3	20	12	9	7	20	19	17	15	19	20	20	20	20	11	3	0
4	20	11	8	7	20	18	16	14	18	20	20	20	20	14	2	0
5	20	11	7	7	20	20	18	17	20	20	20	20	20	17	3	0
6	20	13	8	7	20	20	20	17	20	20	20	20	20	20	2	0
7	20	12	9	7	20	20	15	14	20	20	20	20	20	16	2	0
8	20	13	9	7	20	20	15	13	20	20	20	20	20	13	4	0
9	20	10	7	6	20	18	17	17	18	20	20	20	20	11	2	0
10	20	10	8	7	20	18	15	15	17	17	20	20	20	12	3	0
11	20	11	9	7	20	20	19	12	20	20	20	20	20	14	3	0
12	20	10	8	6	20	19	15	13	18	20	20	20	20	11	2	0
13	20	10	9	6	20	20	18	16	19	20	20	20	20	12	2	0
14	20	10	9	6	20	20	15	14	20	20	20	20	20	10	2	0
15	20	10	8	7	20	20	20	20	19	20	20	20	20	11	2	0
16	20	9	7	7	20	17	15	11	15	17	20	20	20	12	2	0
17	20	10	8	6	20	20	19	19	20	20	20	20	20	19	3	0
18	20	11	8	7	20	16	14	12	15	17	20	20	20	10	3	0
19	20	11	8	7	20	20	20	20	20	20	20	20	20	16	3	0
20	20	13	8	7	20	18	16	13	20	20	20	20	20	10	2	0
21	20	10	8	6	20	20	20	19	20	20	20	20	20	13	3	0
22	20	9	7	6	20	17	16	15	19	19	20	20	20	12	4	0
23	20	12	9	7	20	18	15	18	19	20	20	20	20	11	3	0
24	20	11	9	7	20	20	20	20	20	20	20	20	20	15	2	0
25	20	10	8	6	20	17	15	13	13	18	20	20	20	12	2	0
26	20	11	8	7	20	19	17	16	19	20	20	20	20	13	3	0
27	20	11	8	7	20	20	20	20	20	20	20	20	20	14	3	0

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inst	configuration															
	$\beta=1, \rho=0.02, m=50$				$\alpha=1, \rho=0.02, m=50$				$\alpha=1, \beta=1, \rho=0.02$				$\alpha=1, \beta=1, m=50$			
	$\alpha=$				$\beta=$				$m=$				$\rho=$			
	1	2	3	4	1	2	3	4	20	30	50	70	0.02	0.05	0.2	0.7
28	20	12	9	6	20	20	20	18	20	20	20	20	20	18	3	0
29	20	11	9	6	20	18	20	18	20	20	20	20	20	13	3	0
30	20	10	8	7	20	16	14	12	13	18	20	20	20	11	2	0
31	20	11	8	6	20	19	15	15	15	19	20	20	20	12	3	0
32	20	11	7	6	20	18	15	18	18	20	20	20	20	13	3	0
33	20	10	8	6	20	20	20	14	18	20	20	20	20	12	2	0
34	20	10	9	7	20	19	17	16	17	19	20	20	20	11	2	0
35	20	13	9	7	20	20	18	20	17	20	20	20	20	15	2	0
36	20	13	8	7	20	20	20	17	20	20	20	20	20	15	3	0
37	20	12	8	7	20	20	18	16	20	20	20	20	20	13	2	0
38	20	11	8	6	20	20	19	20	17	20	20	20	20	12	3	0
39	20	12	7	7	20	20	20	19	20	20	20	20	20	12	3	0
40	20	11	7	7	20	20	17	15	15	20	20	20	20	9	2	0
41	20	12	8	7	20	20	16	13	20	19	20	20	20	10	2	0
42	20	11	8	7	20	20	20	17	16	20	20	20	20	14	2	0
43	20	11	8	6	20	20	17	16	20	20	20	20	20	12	3	0
44	20	11	8	7	20	18	15	15	17	20	20	20	20	11	2	0
45	20	11	8	6	20	20	15	16	16	20	20	20	20	12	3	0
46	20	13	10	7	20	20	20	20	20	20	20	20	20	20	2	0
47	20	11	9	6	20	18	15	14	16	20	20	20	20	15	2	0
48	20	11	9	7	20	19	15	17	20	20	20	20	20	12	2	0
49	20	11	8	7	20	20	19	19	17	19	20	20	20	14	3	0
50	20	11	8	7	20	20	19	14	19	20	20	20	20	12	3	0
51	20	10	9	7	20	20	17	15	20	20	20	20	20	16	2	0
52	20	11	8	6	20	17	14	11	12	17	20	20	20	11	4	0
53	20	11	8	6	20	20	15	14	20	20	20	20	20	11	2	0
54	20	12	7	6	20	20	18	13	20	20	20	20	20	12	2	0
55	20	11	8	6	20	20	17	17	20	20	20	20	20	14	2	0
56	20	10	8	7	20	17	18	15	19	20	20	20	20	12	2	0
57	20	11	8	6	20	17	20	13	17	20	20	20	20	14	3	0
58	20	10	7	7	20	20	18	18	15	20	20	20	20	11	3	0
59	20	13	8	6	20	20	20	20	19	20	20	20	20	16	3	0
60	20	11	9	7	20	20	19	17	16	20	20	20	20	15	3	0
61	20	11	9	7	20	20	20	17	20	20	20	20	20	13	3	0
62	20	13	8	6	20	20	19	20	20	20	20	20	20	13	3	0
63	20	12	9	6	20	20	20	17	20	20	20	20	20	14	2	0
64	20	11	7	6	20	19	17	16	17	20	20	20	20	10	2	0
65	20	11	9	7	20	19	18	19	17	20	20	20	20	11	3	0
66	20	12	8	7	20	20	20	18	20	20	20	20	20	18	2	0
67	20	11	9	7	20	20	16	14	20	20	20	20	20	13	2	0
68	20	11	8	6	20	17	18	16	20	20	20	20	20	12	3	0
69	20	10	9	7	20	20	20	20	17	20	20	20	20	14	2	0
70	20	11	7	6	20	19	17	15	15	20	20	20	20	10	2	0
71	20	10	7	7	20	20	16	16	17	17	20	20	20	11	2	0
72	20	13	8	7	20	20	18	18	20	20	20	20	20	15	3	0
73	20	10	8	6	20	20	18	18	15	19	20	20	20	11	3	0
74	20	12	8	7	20	18	16	16	17	20	20	20	20	17	2	0
75	20	11	9	7	20	20	20	20	20	20	20	20	20	14	3	0
76	20	11	9	8	20	20	19	16	19	20	20	20	20	16	4	0
77	20	11	8	6	20	20	19	17	20	20	20	20	20	13	3	0
78	20	11	7	7	20	19	16	16	14	18	20	20	20	11	2	0
79	20	11	8	6	20	20	20	19	19	20	20	20	20	15	2	0
80	20	11	8	7	20	20	19	17	20	20	20	20	20	12	2	0
81	20	11	8	7	20	17	13	13	20	20	20	20	20	12	2	0
82	20	12	9	7	20	20	16	15	14	20	20	20	20	14	3	0
83	20	12	8	7	20	20	19	20	20	20	20	20	20	13	2	0
84	20	12	10	7	20	20	19	15	15	20	20	20	20	12	2	0
85	20	11	9	7	20	20	17	18	20	20	20	20	20	13	2	0
86	20	11	8	8	20	20	17	16	18	20	20	20	20	12	3	0
87	20	11	9	6	20	18	13	13	15	18	20	20	20	13	2	0
88	20	11	8	6	20	20	20	20	19	20	20	20	20	11	2	0
89	20	11	8	7	20	20	20	19	20	20	20	20	20	15	2	0
90	20	12	8	8	20	20	19	18	15	19	20	20	20	13	2	0
91	20	10	8	7	20	20	19	20	20	20	20	20	20	18	3	0
92	20	10	7	7	20	14	12	10	11	15	20	20	20	9	2	0
93	20	9	8	6	20	20	16	16	14	17	20	20	20	12	3	0
94	20	11	8	6	20	20	18	14	19	20	20	20	20	10	2	0
95	20	11	8	6	20	18	15	15	19	20	20	20	20	12	3	0
96	20	11	8	8	20	20	15	14	16	20	20	20	20	12	2	0
97	20	12	8	6	20	20	18	16	20	20	20	20	20	11	3	0
98	20	11	8	8	20	19	16	14	20	20	20	20	20	11	2	0
99	20	11	8	7	20	20	17	17	20	20	20	20	20	17	2	0
100	20	13	8	8	20	20	18	20	18	20	20	20	20	10	3	0

The results obtained for all the 100 instances are summarized in Table 1. The number of objective function evaluations (thousands) after which the exploration reaches its maximum value is reported. Moreover, the corresponding value of exploration is indicated between parenthesis. As it can be remarked, the trend observed in figures 1(a), 1(c), 1(e), 1(g) (instance 1) are clearly respected.

In parallel, in Table 2 the number of objective function evaluations (thousands) after which the average λ-branching factor becomes lower than 3 is indicated.

Figures 3(a), 3(c), 3(e) and 3(g) represent the objective function value associated to the best-so-far solution for the different values of the parameters. Figures 3(b), 3(d), 3(f) and 3(h) describe the ratio of the value of the best-so-far solution generated by *MAX–MIN* Ant System and a random search algorithm. It is interesting to notice that only when exploration passes the *random–like* phase the ACO algorithm behaves in a way that is significantly different from the random search. This is true for any value of the parameters.

Table 3 Thousands of objective function evaluations after which the value of the best so far solution found by *MM*AS is lower than 40% of the one found by a random search.

inst	configuration: $\beta=1, \rho=0.02, m=50$, $\alpha=$				$\alpha=1, \rho=0.02, m=50$, $\beta=$				$\alpha=1, \beta=1, \rho=0.02$, $m=$				$\alpha=1, \beta=1, m=50$, $\rho=$			
	1	2	3	4	1	2	3	4	20	30	50	70	0.02	0.05	0.2	0.7
1	14	6	4	4	14	9	5	0	20	20	14	20	14	6	2	2
2	9	5	4	3	9	3	0	0	11	9	9	12	9	3	1	0
3	13	7	5	4	13	8	3	0	15	14	13	16	13	6	2	3
4	9	6	4	3	9	4	0	0	9	10	9	13	9	4	1	1
5	9	5	4	3	9	4	0	0	11	9	9	12	9	4	1	0
6	14	7	5	4	14	7	0	0	14	13	14	20	14	6	2	3
7	13	6	5	4	13	8	3	0	20	14	13	20	13	7	2	2
8	10	6	5	4	10	7	3	0	14	12	10	19	10	5	2	1
9	10	5	4	3	10	4	0	0	9	9	10	12	10	4	1	0
10	11	6	4	3	11	5	0	0	11	12	11	17	11	4	1	2
11	20	7	6	5	20	12	8	3	20	20	20	20	20	8	3	6
12	14	6	5	4	14	9	7	0	20	15	14	20	14	5	2	8
13	15	7	6	4	15	10	7	4	19	18	15	20	15	8	2	3
14	8	5	4	3	8	2	0	0	9	8	8	12	8	3	1	0
15	16	7	5	4	16	9	7	0	15	16	16	19	16	6	2	2
16	9	6	4	3	9	3	0	0	11	11	9	13	9	4	1	1
17	13	6	5	4	13	6	0	0	13	12	13	19	13	5	2	1
18	11	6	4	4	11	5	0	0	11	10	11	13	11	4	1	1
19	9	6	4	3	9	5	0	0	10	8	9	13	9	4	1	1
20	14	7	5	4	14	8	0	0	20	18	14	20	14	6	2	2
21	10	6	5	4	10	6	1	0	13	12	10	19	10	5	1	1
22	10	6	4	3	10	5	0	0	11	11	10	14	10	4	1	1
23	10	6	4	4	10	6	0	0	10	9	10	14	10	4	1	0
24	10	6	4	3	10	4	0	0	10	9	10	13	10	4	1	0
25	9	4	4	3	9	2	0	0	9	9	9	11	9	3	1	1
26	12	7	5	4	12	7	0	0	12	12	12	16	12	5	1	1
27	20	7	5	4	20	10	6	0	20	20	20	20	20	7	2	2
28	14	7	6	4	14	10	7	4	13	20	14	20	14	7	2	4
29	14	7	5	4	14	9	5	0	20	14	14	20	14	5	2	2
30	8	5	4	3	8	0	0	0	8	9	8	13	8	3	1	0
31	14	6	5	4	14	7	3	0	15	14	14	19	14	5	2	1
32	10	5	4	3	10	0	0	0	10	10	10	13	10	4	1	1
33	11	6	4	3	11	7	0	0	11	12	11	17	11	4	1	1
34	12	7	5	4	12	8	0	0	14	13	12	19	12	7	2	2
35	12	7	5	4	12	8	5	1	18	14	12	19	12	6	2	2
36	12	7	5	4	12	9	3	0	16	17	12	20	12	5	1	1
37	20	7	5	4	20	10	6	1	20	20	20	20	20	5	2	5
38	11	6	4	3	11	6	0	0	12	11	11	15	11	5	1	1
39	12	7	5	4	12	7	2	0	14	11	12	16	12	5	3	1
40	9	5	4	3	9	0	0	0	10	10	9	12	9	4	1	0
41	13	7	5	4	13	8	2	0	14	14	13	20	13	5	2	1
42	12	6	5	4	12	7	1	0	13	13	12	20	12	5	2	1
43	12	6	4	4	12	6	0	0	13	12	12	19	12	5	2	1
44	13	6	5	4	13	7	1	0	13	12	13	20	13	5	1	1
45	20	8	5	5	20	12	7	8	20	20	20	20	20	7	3	8
46	20	9	6	5	20	14	11	9	20	20	20	20	20	9	3	5
47	9	5	4	3	9	5	0	0	10	10	9	14	9	4	1	1
48	9	5	4	3	9	4	0	0	10	10	9	12	9	4	1	1
49	12	6	4	3	12	6	0	0	13	12	12	16	12	5	2	2
50	13	6	5	4	13	8	4	0	16	15	13	18	13	5	2	3
51	14	7	5	4	14	9	2	0	16	13	14	17	14	6	2	1

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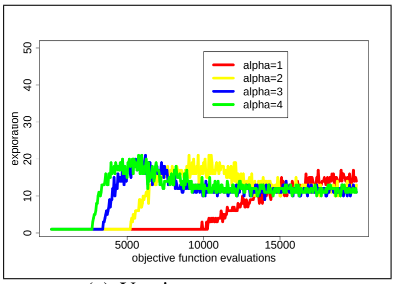
inst	configuration															
	$\beta=1,\rho=0.02,m=50$				$\alpha=1,\rho=0.02,m=50$				$\alpha=1,\beta=1,\rho=0.02$				$\alpha=1,\beta=1,m=50$			
	$\alpha=$				$\beta=$				$m=$				$\rho=$			
	1	2	3	4	1	2	3	4	20	30	50	70	0.02	0.05	0.2	0.7
52	10	6	4	3	10	4	0	0	10	10	10	14	10	4	1	0
53	10	5	4	3	10	3	0	0	9	10	10	14	10	4	1	1
54	10	6	4	4	10	6	0	0	10	11	10	13	10	4	1	1
55	9	6	4	4	9	5	0	0	11	11	9	14	9	5	1	2
56	11	6	5	4	11	7	0	0	13	12	11	16	11	5	1	1
57	10	5	4	3	10	2	0	0	11	10	10	12	10	4	1	0
58	9	5	4	3	9	0	0	0	10	10	9	13	9	3	1	1
59	11	6	4	4	11	6	0	0	13	12	11	14	11	5	1	1
60	14	7	6	4	14	9	8	2	18	20	14	20	14	6	2	2
61	16	7	6	5	16	8	3	0	18	14	16	19	16	7	2	3
62	12	6	5	4	12	7	0	0	14	13	12	17	12	5	2	1
63	18	7	5	4	18	12	7	0	18	18	18	20	18	6	3	5
64	11	6	4	4	11	7	2	0	12	12	11	18	11	5	1	1
65	8	5	4	3	8	3	0	0	10	10	8	12	8	4	1	1
66	12	6	5	4	12	8	4	0	14	13	12	20	12	5	2	2
67	10	6	4	3	10	5	0	0	13	11	10	14	10	4	2	1
68	9	5	4	3	9	2	0	0	10	10	9	12	9	4	1	1
69	11	6	4	4	11	5	0	0	11	11	11	15	11	4	1	0
70	9	6	4	3	9	6	0	0	11	12	9	14	9	4	1	1
71	9	5	4	4	9	5	0	0	11	11	9	12	9	4	1	1
72	20	8	6	5	20	12	10	0	20	20	20	20	20	8	2	3
73	8	6	4	3	8	3	0	0	11	10	8	13	8	4	1	1
74	11	6	5	4	11	6	0	0	12	11	11	18	11	5	1	1
75	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
76	20	8	7	7	20	14	9	2	20	20	20	20	20	12	3	7
77	19	7	5	4	19	10	7	3	20	20	19	20	19	7	6	5
78	10	6	4	3	10	5	0	0	12	10	10	14	10	4	1	0
79	14	7	5	4	14	10	8	3	20	20	14	20	14	7	2	1
80	14	6	5	4	14	7	1	0	19	15	14	17	14	7	1	1
81	14	6	5	4	14	7	1	0	16	12	14	17	14	5	2	1
82	12	6	5	4	12	7	3	0	13	13	12	19	12	5	2	1
83	20	8	6	5	20	13	9	8	20	20	20	20	20	11	2	12
84	11	6	5	4	11	6	0	0	12	12	11	19	11	4	1	2
85	12	6	5	4	12	7	1	0	12	11	12	16	12	5	1	1
86	10	6	5	4	10	5	0	0	13	12	10	14	10	5	1	1
87	10	6	4	3	10	6	0	0	12	10	10	12	10	4	1	1
88	10	6	4	3	10	3	0	0	11	9	10	14	10	4	1	1
89	12	6	5	4	12	6	0	0	10	12	12	14	12	5	1	2
90	12	6	5	4	12	8	4	0	12	13	12	17	12	5	1	2
91	17	8	5	4	17	11	10	6	20	20	17	20	17	10	3	2
92	8	6	4	3	8	0	0	0	9	10	8	13	8	4	1	0
93	10	6	4	3	10	4	0	0	10	10	10	14	10	4	1	0
94	9	5	4	3	9	4	0	0	12	11	9	14	9	4	1	1
95	16	7	5	4	16	7	5	3	17	17	16	20	16	6	2	3
96	10	6	4	4	10	6	0	0	12	11	10	16	10	5	1	1
97	13	6	5	4	13	6	2	0	14	13	13	16	13	5	2	1
98	8	5	4	3	8	4	0	0	10	8	8	13	8	4	1	1
99	9	5	3	3	9	4	0	0	10	10	9	14	9	4	1	1
100	14	8	5	5	14	10	7	0	14	20	14	20	14	6	2	4

Table 3 indicates, for all the 100 instances, the number of objective function evaluations (thousands) after which the value of the best so far solution found by *MAX–MIN* Ant System is lower than 40% of the one found by a random search.

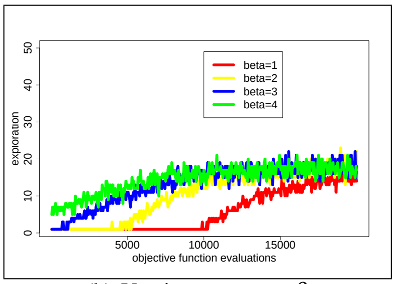
Appendix 1

The results reported in Section 1 are obtained using the Euclidean distance between solutions. Then the number of clusters is computed for the different parameter settings. Here the same computation is presented, but based on two different distance measures, namely the Manhattan distance and the p-distance, with p=3. The value of ε_{10} varies consequently: it is 20 and 2.15, respectively. One instance is considered.

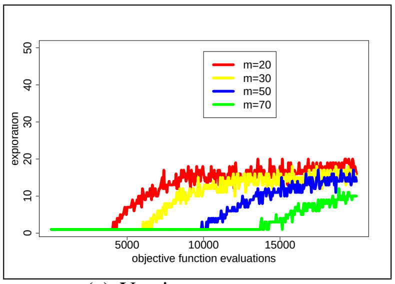
The results are reported in Figures 4 and 5 respectively. The number of objective function evaluations is reported on the x-axis, while the number of clusters is on the y-axis.



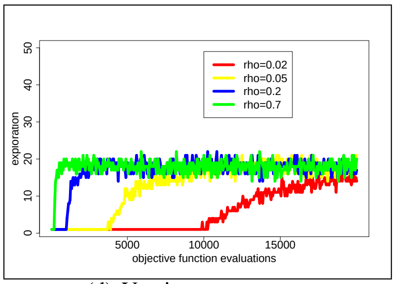
(a) Varying parameter α



(b) Varying parameter β



(c) Varying parameter m



(d) Varying parameter ρ

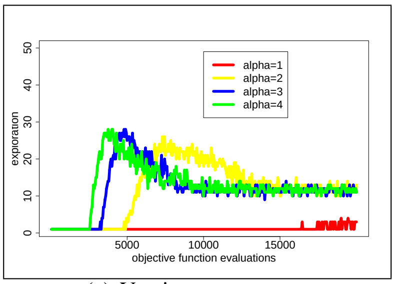
Fig. 4 Number of clusters computed using the Manhattan distance

As it can be observed, the conclusions that can be drawn are qualitatively the same in the three cases. This observation allows to think that the specific distance used does not affects significantly the results reported, at least for what their general meaning is concerned.

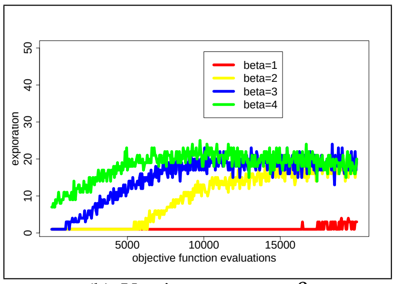
Appendix 2

The results reported above are obtained considering instances with 50 nodes. Here parallel results are presented (Figures 6, 7, 8, 9), based in instances with 100 and 200 nodes. The Euclidean distance is used, and the consequence values of ε_{10} are 4.47 and 6.32, respectively. One instance is considered.

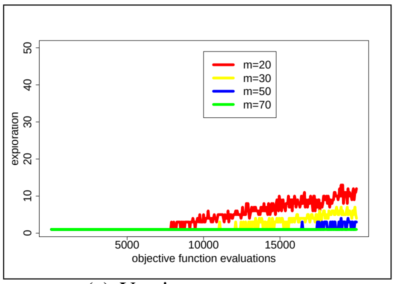
As it can be observed, the conclusions that can be drawn are qualitatively the same despite the change in the size of the instance.



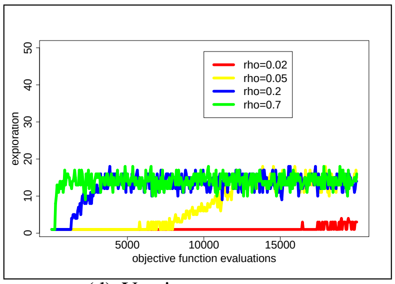
(a) Varying parameter α



(b) Varying parameter β

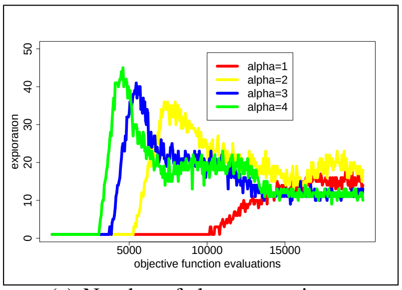


(c) Varying parameter m

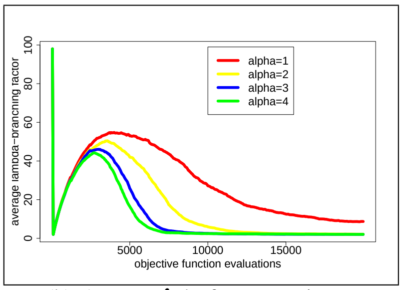


(d) Varying parameter ρ

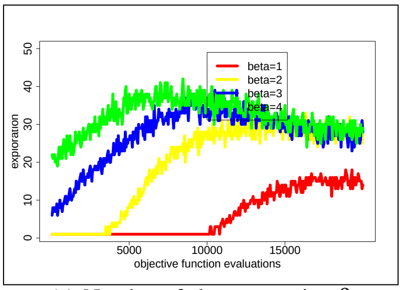
Fig. 5 Number of clusters computed using the p-distance (p=3)



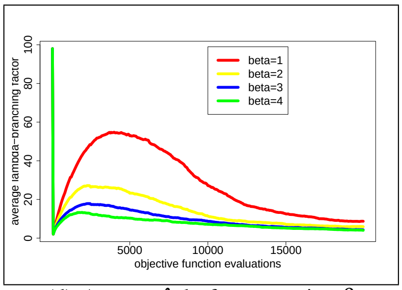
(a) Number of clusters varying α



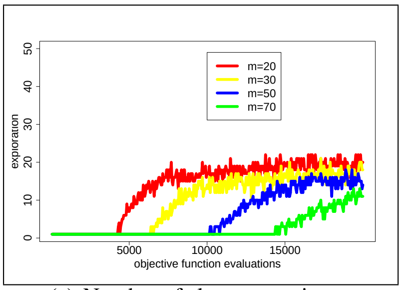
(b) Average λ-br factor varying α



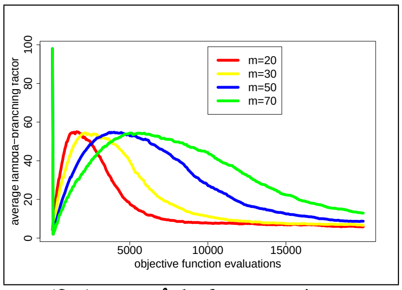
(c) Number of clusters varying β



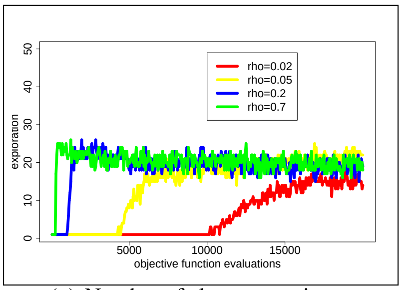
(d) Average λ-br factor varying β



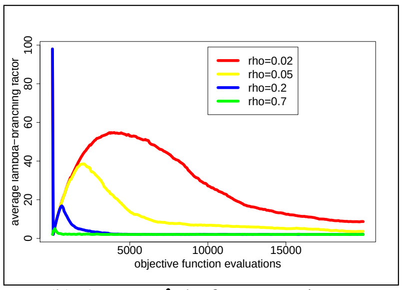
(e) Number of clusters varying m



(f) Average λ-br factor varying m

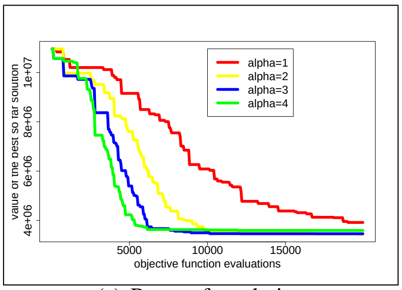


(g) Number of clusters varying ρ

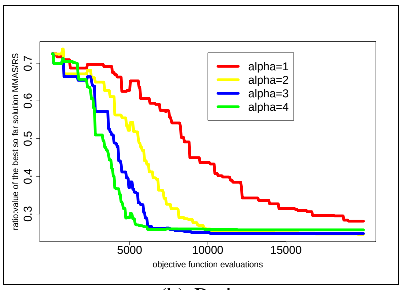


(h) Average λ-br factor varying ρ

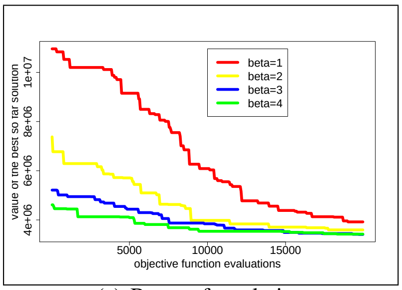
Fig. 6 Number of clusters versus average λ-branching factor (instance with 100 nodes)



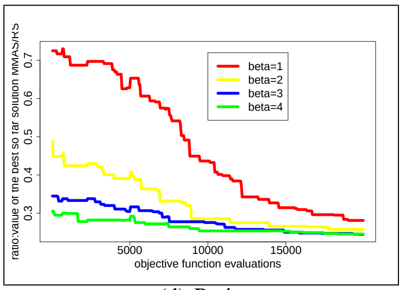
(a) Best so far solution



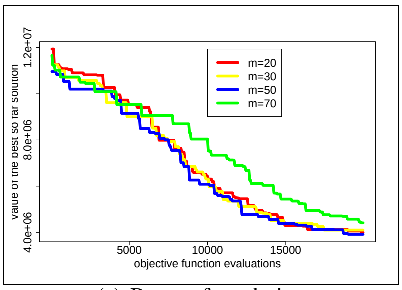
(b) Ratio



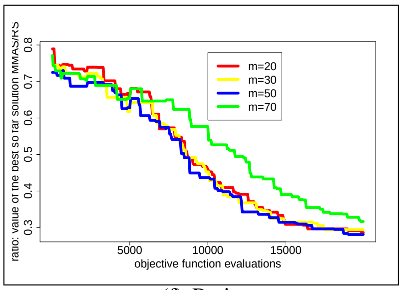
(c) Best so far solution



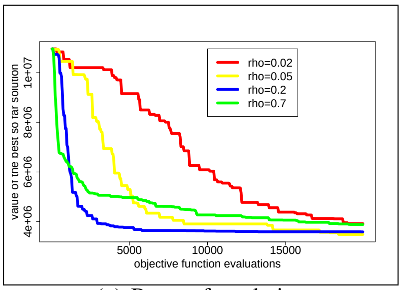
(d) Ratio



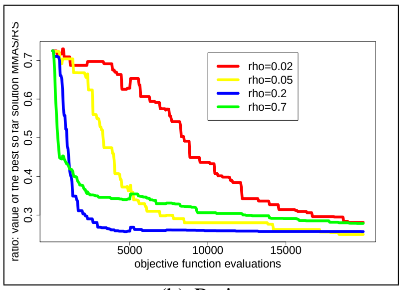
(e) Best so far solution



(f) Ratio

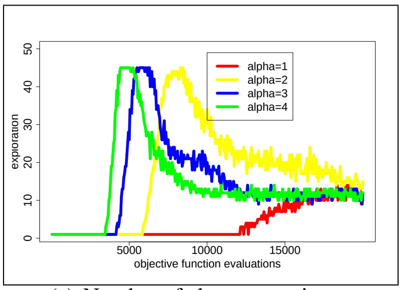


(g) Best so far solution

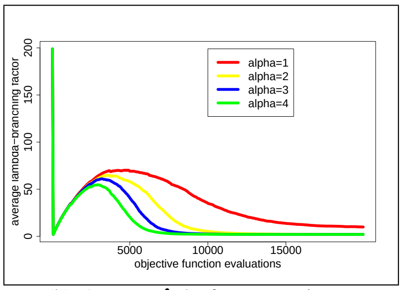


(h) Ratio

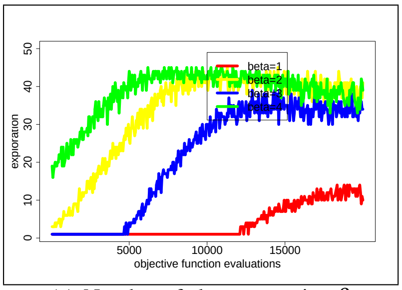
Fig. 7 Best so far solution and ratio with respect to random search (instance with 100 nodes)



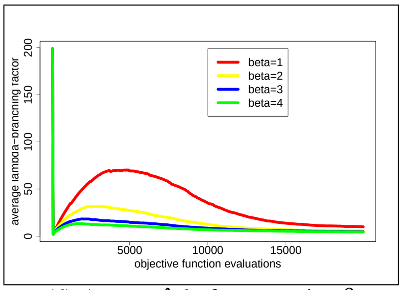
(a) Number of clusters varying α



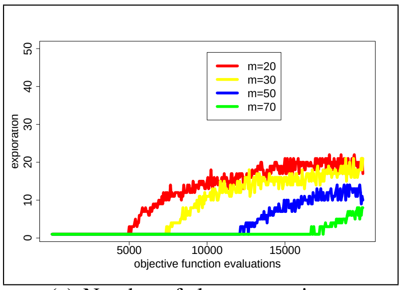
(b) Average λ-br factor varying α



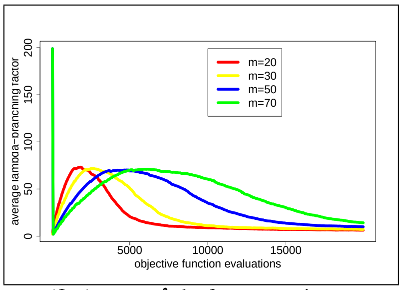
(c) Number of clusters varying β



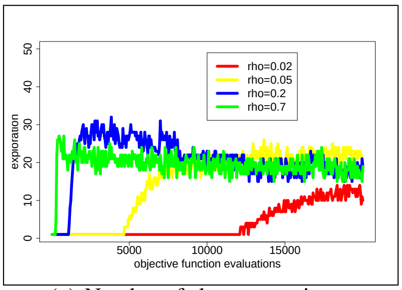
(d) Average λ-br factor varying β



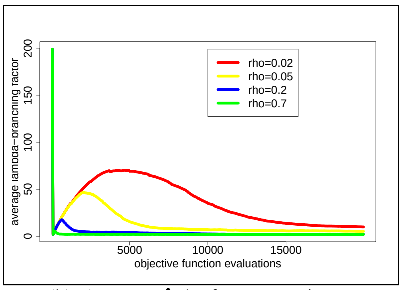
(e) Number of clusters varying m



(f) Average λ-br factor varying m

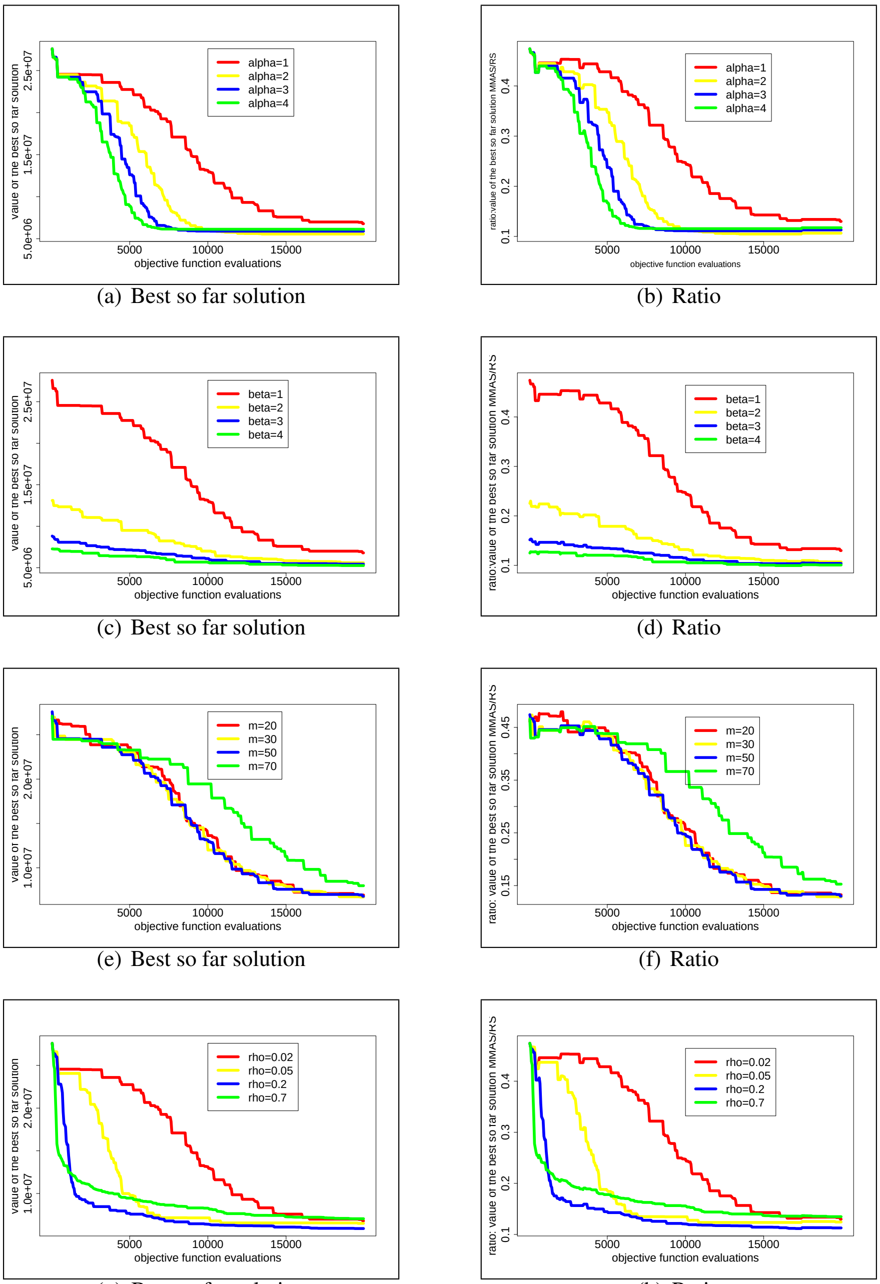


(g) Number of clusters varying ρ



(h) Average λ-br factor varying ρ

Fig. 8 Number of clusters versus average λ-branching factor (instance with 200 nodes)



(a) Best so far solution (b) Ratio

(c) Best so far solution (d) Ratio

(e) Best so far solution (f) Ratio

(g) Best so far solution (h) Ratio

Fig. 9 Best so far solution and ratio with respect to random search (instance with 200 nodes)